Topic: Completing a right triangle

Question: What is the measure, in radians, of the acute interior angle that's opposite the other leg?

Consider a right triangle. If the measure, in radians, of the acute interior angle which is opposite one of the legs of the right triangle is $(2/5)\pi$, what is the measure, in radians, of the acute interior angle that's opposite the other leg?

Answer choices:

A $\frac{\pi}{10}$

B $\frac{6\pi}{5}$

C $\frac{3\pi}{5}$

D $\frac{11\pi}{10}$

© Krista King Math

Solution: A

Let θ be the given angle (i.e., $\theta = (2/5)\pi$), and let α be the acute interior angle which is opposite the other leg of the right triangle.

In degrees, the sum of the measures of θ and α is $90°$. Since $180°$ is equivalent to π radians, we see that $90°$ is equivalent to $\pi/2$ radians. Thus

$$\theta + \alpha = \frac{\pi}{2}$$

Substituting the value of θ, we have

$$\frac{2}{5}\pi + \alpha = \frac{\pi}{2}$$

Solving for α:

$$\alpha = \frac{\pi}{2} - \frac{2}{5}\pi$$

$$\alpha = \frac{5(\pi) - 2(2\pi)}{10}$$

$$\alpha = \frac{5\pi - 4\pi}{10}$$

$$\alpha = \frac{\pi}{10}$$

© Krista King Math

Topic: Completing a right triangle

Question: Which of the following most closely approximates the length of the other leg of that right triangle?

Suppose the length of one of the legs of a right triangle is 5 and the measure of the acute interior angle opposite that leg is $58°$.

Answer choices:

A 2.65

B 8.00

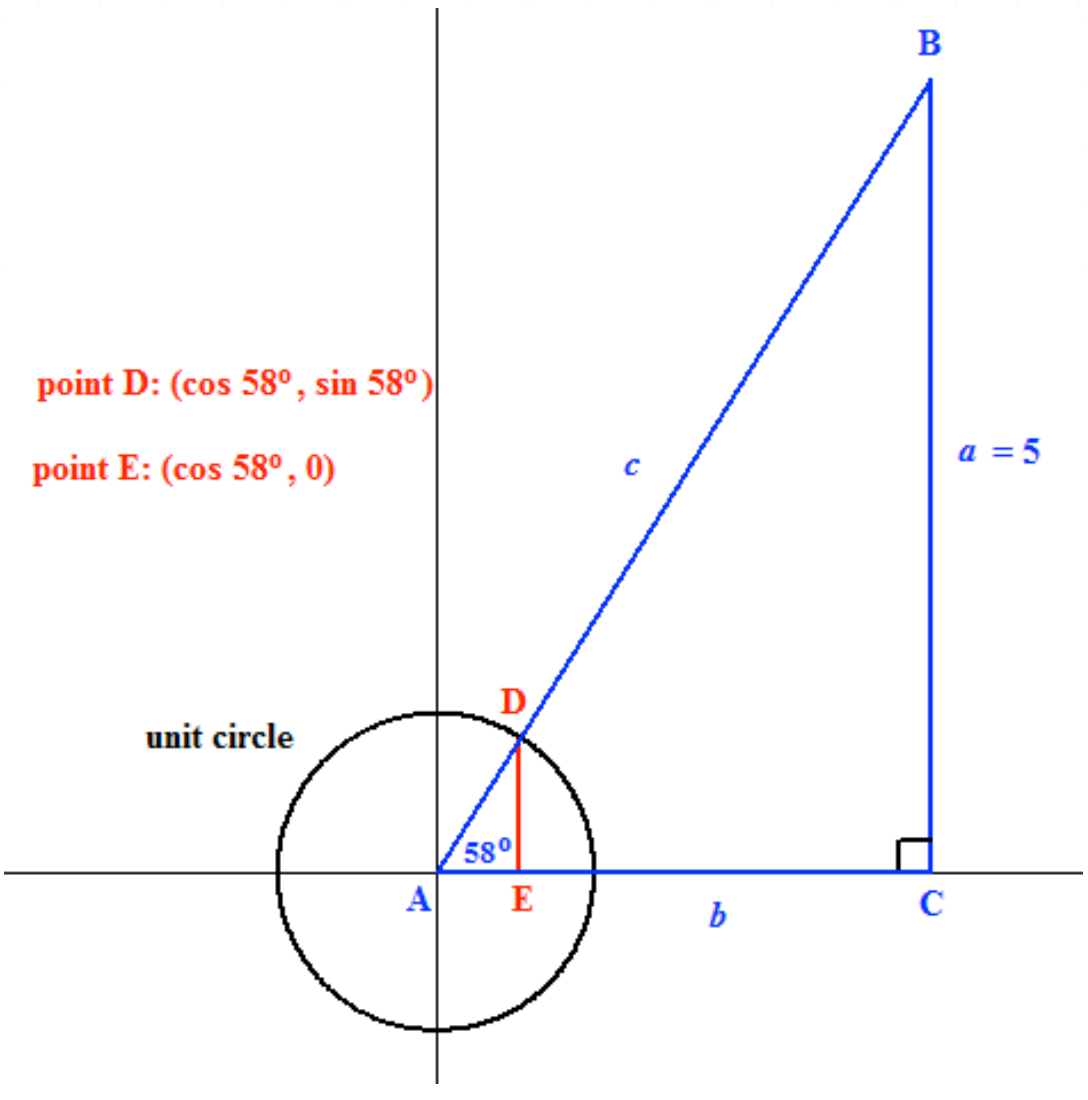
C 4.24

D 3.13

© Krista King Math

Solution: D

We'll draw this right triangle (triangle ABC) with vertex A at the origin, vertex B in the first quadrant, and vertex C on the positive x-axis. We'll let BC be the leg whose length we've been given, and we'll let a be the length of that leg (i.e., the length of side BC is $a = 5$). Then the measure of the acute interior angle which is opposite that leg (i.e., angle BAC) is $58°$. We'll let b be the length of the other leg (side AC) and c the length of the hypotenuse (side AB). Then we'll draw a new right triangle (triangle ADE) which is similar to ABC, with the sides of triangle ADE oriented along the corresponding sides of triangle ABC and the "outer" endpoint of the hypotenuse (point D) on the unit circle.



By the similarity of triangles ABC and ADE, we have the following:

© Krista King Math

$$\frac{a}{\overline{DE}} = \frac{b}{\overline{AE}}$$

Substituting $a = 5$, we have

$$\frac{5}{\overline{DE}} = \frac{b}{\overline{AE}}$$

Since point D is on the unit circle, its coordinates (x, y) are $(\cos 58°, \sin 58°)$. Also, the coordinates (x, y) of point E are $(\cos 58°, 0)$.

Note that DE is parallel to the positive y-axis, so the length of DE is equal to the difference between the y-coordinate of point D (which is $\sin 58°$) and the y-coordinate of point E (which is 0):

$$\overline{DE} = \sin 58° - 0 = \sin 58°$$

Also, the length of side AE is equal to the difference between the x-coordinate of point E (which is $\cos 58°$) and the x-coordinate of point A (which is 0):

$$\overline{AE} = \cos 58° - 0 = \cos 58°$$

Substituting these results, we find that

$$\frac{5}{\sin 58°} = \frac{b}{\cos 58°}$$

To solve for b (the length of AC), we multiply both sides of this equation by $\cos 58°$:

$$\left(\frac{5}{\sin 58°}\right)(\cos 58°) = b$$

© Krista King Math

Turning this equation around, we obtain

$$b = \left(\frac{5}{\sin 58^\circ}\right)(\cos 58^\circ)$$

Using a calculator, we can compute $\cos 58^\circ$ and $\sin 58^\circ$:

$$\sin 58^\circ \approx 0.848$$

$$\cos 58^\circ \approx 0.530$$

Thus

$$b \approx \left(\frac{5}{0.848}\right)(0.530) \approx 3.13$$

© Krista King Math

Topic: Completing a right triangle

Question: Which of the following most closely approximates the length of the hypotenuse of the right triangle?

The length of one of the legs is 2.4 and the measure of the acute interior angle which is opposite the other leg is $28°$.

Answer choices:

A 2.72

B 4.51

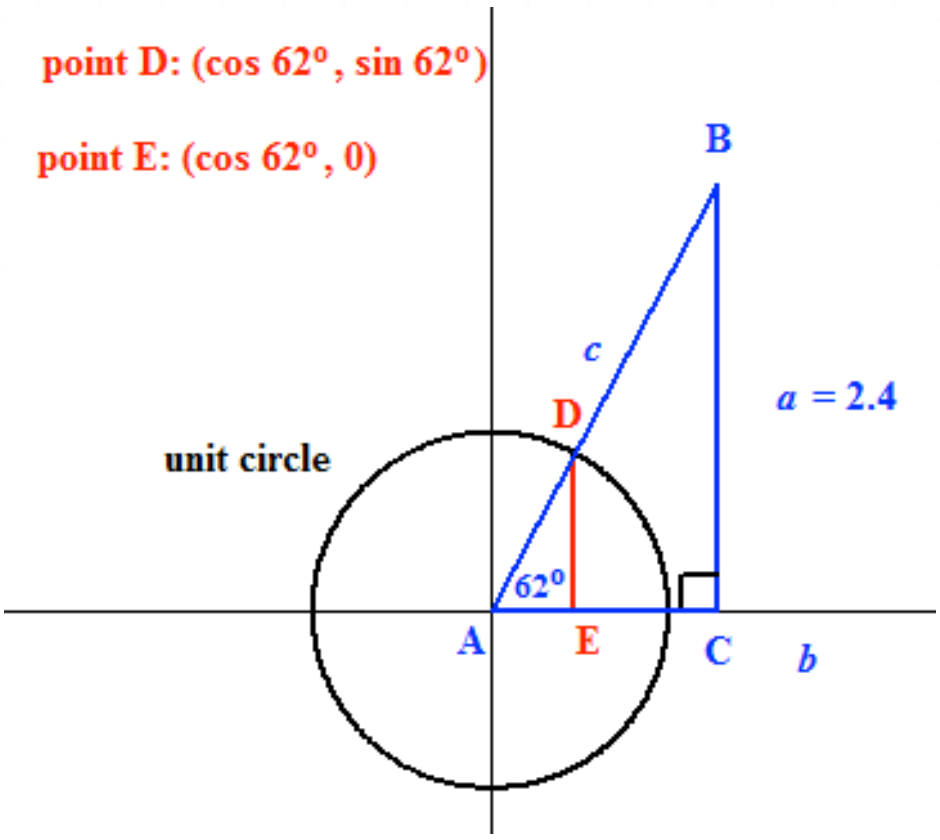
C 1.28

D 5.11

© Krista King Math

Solution: A

We'll draw this right triangle (triangle ABC) with vertex A at the origin, vertex B in the first quadrant, and vertex C on the positive x-axis. We'll let BC be the leg whose length is known, and we'll let a be the length of that leg (i.e., the length of side BC is $a = 2.4$). Then the measure of the acute interior angle which is opposite that leg (i.e., angle BAC) is $90^\circ - 28^\circ = 62^\circ$. We'll let b be the length of the other leg (side AC) and c the length of the hypotenuse (side AB). Then we'll draw a new right triangle (triangle ADE) which is similar to ABC, with the sides of triangle ADE oriented along the corresponding sides of triangle ABC and the "outer" endpoint of the hypotenuse (point D) on the unit circle.



By the similarity of triangles ABC and ADE, we have the following:

$$\frac{a}{\overline{DE}} = \frac{c}{\overline{AD}}$$

Substituting $a = 2.4$, we have

© Krista King Math

$$\frac{2.4}{\overline{DE}} = \frac{c}{\overline{AD}}$$

Since point D is on the unit circle, its coordinates (x, y) are $(\cos 62^\circ, \sin 62^\circ)$ and the length of AD is 1. Also, the coordinates (x, y) of point E are $(\cos 62^\circ, 0)$.

Note that DE is parallel to the positive y-axis, so the length of DE is equal to the difference between the y-coordinate of point D (which is $\sin 62^\circ$) and the y-coordinate of point E (which is 0):

$$\overline{DE} = \sin 62^\circ - 0 = \sin 62^\circ$$

Substituting these results, we find that

$$\frac{2.4}{\sin 62^\circ} = \frac{c}{1}$$

Simplifying the right-hand side and then turning this equation around, we have

$$c = \frac{2.4}{\sin 62^\circ}$$

Using a calculator, we can compute $\sin 62^\circ$:

$$\sin 62^\circ \approx 0.883$$

Thus

$$c \approx \frac{2.4}{0.883}$$

$$c \approx 2.72$$

© Krista King Math